

# Extinction Magnet Specifications for the Mu2e Experiment

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## Abstract

This note presents preliminary specifications for magnet design in several possible configurations which have been proposed for the extinction system in the Mu2e experiment. General optimization of the design drives us to long, low field magnets in regions of high  $\beta$  in the bend plane, and we have used  $\beta = 250$  m and a 6 m total length for all designs considered.

## 1 Introduction

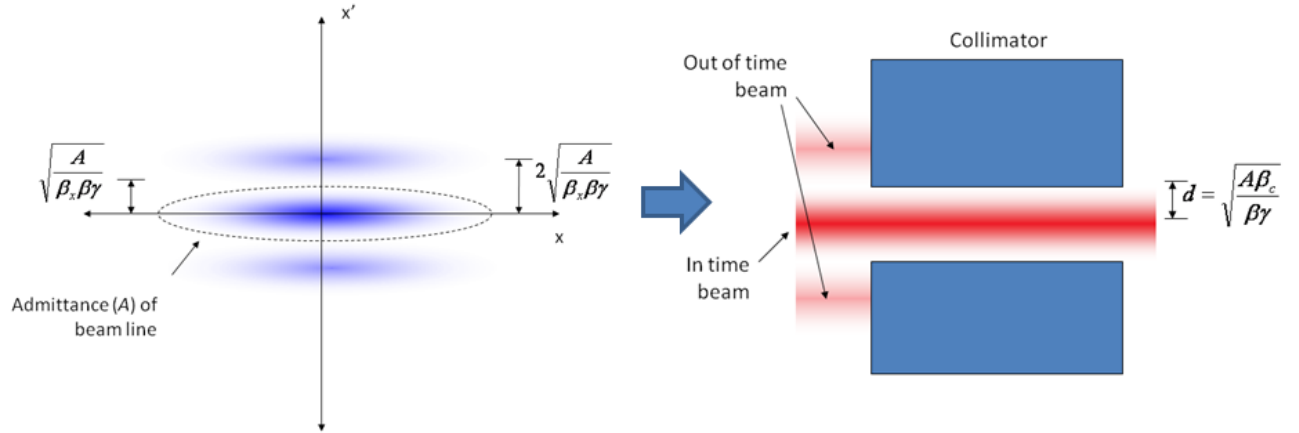


Figure 1: Effect of the extinction magnet in phase space. Beam line admittance  $A$  is indicated by the ellipse. Shown at right is effect of the dipole at the collimator (or other defining aperture).

Elsewhere[1], we have discussed the relationship between requirements of an AC dipole and the local beam line parameters in the context of a single harmonic system operating at half the bunch

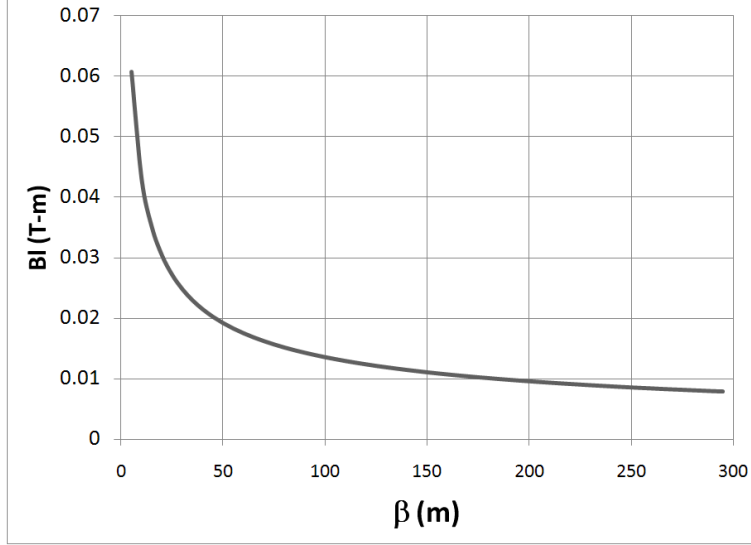


Figure 2: The minimum integrated bend field required for complete extinction as a function of  $\beta_x$ , assuming an admittance of  $50 \pi$ -mm-mr. Note that depending on the required waveform, the actual peak integrated field may be significantly higher.

rate. We present a generalized and simplified version of that argument here. Throughout, we will assume a kinetic energy of 8 GeV and a normalized beam admittance of  $50 \pi$ -mm-mr.

The effect of the extinction magnet/collimator system can be represented as a translation in transverse phase space, as illustrated in Figure 1. In general, out of time beam may be present anywhere within the admittance of the beam line, so complete extinction is not assured until a deflection amplitude of

$$\Delta\theta = 2\sqrt{\frac{A}{\beta_x\beta\gamma}} \quad (1)$$

is achieved, where  $\beta_x$  is the betatron function in the bend plane and  $\beta$  and  $\gamma$  have their usual definitions. We will therefore use this as our definition of the amplitude for complete extinction. The required integrated field achieve extinction is then

$$Bl = 2(B\rho)\sqrt{\frac{A}{\beta_x\beta\gamma}} \quad (2)$$

as plotted in Figure 2 for the nominal Mu2e beam line parameters.

Generally speaking, magnet and power supply complexity increases with stored energy

$$U \propto B^2 L w g = ((Bl)^2 / L) w g \quad (3)$$

Here,  $w$  is the aperture in the bend plane, which is proportional to  $\sqrt{\beta_x}$ . The gap  $g$  is the minimum aperture in the non-bend plane through which the beam can pass. Ignoring chromatic effects, this is proportional to  $\sqrt{L}$ , so we have

$$U \propto \frac{1}{\sqrt{L\beta_x}} \quad (4)$$

Thus, regardless of the details of the magnet design, we are driven toward the highest values of  $\beta_x$  and the longest magnets which can be accommodated in the beam line. Preliminary analysis has indicated that the Mu2e beam line should be able to accommodate a  $\beta_x$  value up to about 250 m and a total length of 6 m[2], so we will adopt these as our working points. Note that this reduces the stored energy by a factor of almost four relative to our preliminary specifications of  $\beta_x=50$  m and  $L=6$  m; however, because of the weak dependence of Eq. 4 on these parameters, significant further improvement would be difficult.

## 2 Pulse Shape (Harmonic Content)

In addition to the total field requirement, one must consider the transmission efficiency of the in time beam, and this is a particular concern for the single harmonic design in the Mu2e proposal. This issue was analyzed in detail in [3]. Several schemes were considered, as illustrated in Figure 3:

- **Sine Wave:** A sine wave running at half the bunch rate, as described in the Mu2e proposal[4].
- **Modified Sine Wave A:** The same sine wave modified by a sine wave with the opposite polarity, 1/17 the amplitude, and 17 times the frequency (5.1 MHz), to reduce the beam slewing to exactly zero at the nominal bunch time.
- **Modified Sine Wave B:** The configuration described above, but with the higher harmonic having 2/17 the amplitude of the primary to expand the transmission window somewhat further.
- **MECO:** The configuration proposed for MECO[5], comprised of magnets operating at the first three harmonics of the bunch frequency, with relative amplitudes of 1:.74:.63, to approximate a short square pulse.

In all cases, the amplitude was normalized to give complete extinction at  $\pm 100$  ns, according to Eq. 1. The transmission results are shown in Figure 4. As can be seen, the single dipole scheme in the proposal has significant beam loss problems for bunch lengths longer than  $\sigma_t \approx 10$  ns.

The “MECO” scheme is superior in this regard, as is “Modified Sine Wave A”. “Modified Sine Wave B” has the longest full transmission window, but can suffer inefficiencies for larger beam emittances due to beam scraping against the opposite wall of the collimator.

In this discussion, we are considering an additional option:

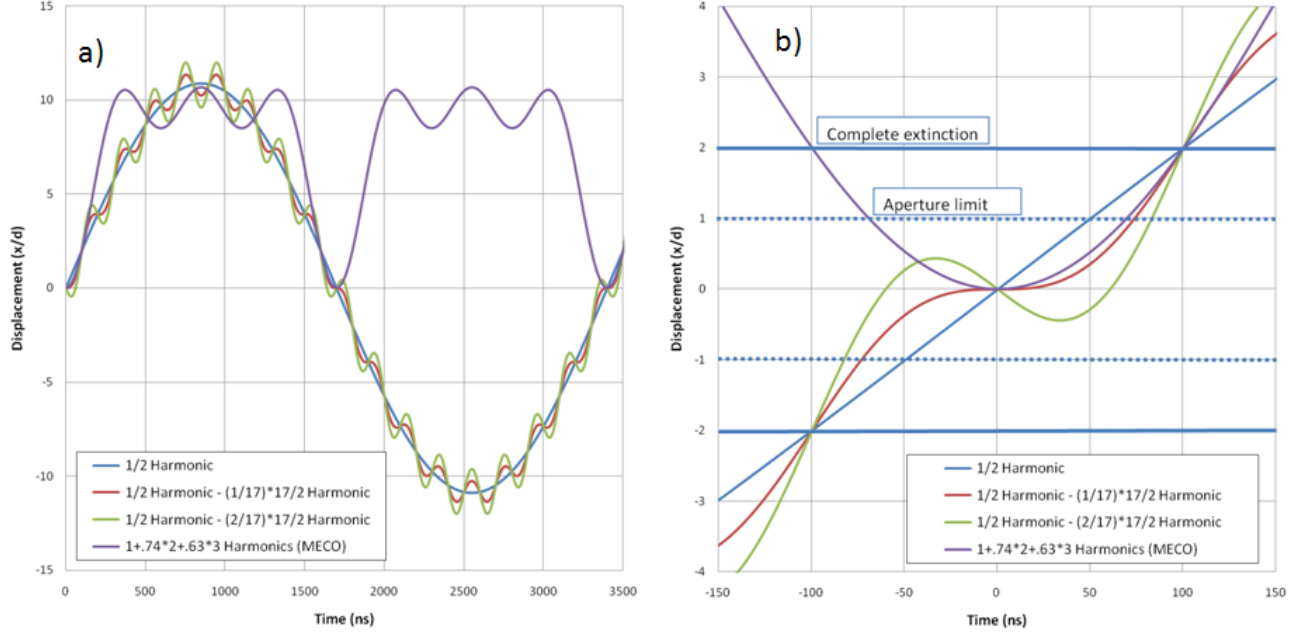


Figure 3: The extinction kicker waveforms which are analyzed in this note. In all cases, amplitudes have been normalized so that complete extinction is achieved at  $\pm 100$  ns. Figure a) shows the complete waveform over two bunch cycles, while b) shows the detail near the nominal bunch time. The defining aperture is indicated, as is the amplitude corresponding to complete extinction. Note that the MECO waveform has been shifted to put the peak at zero, to facilitate comparison with the other designs. In addition to these waveforms, we also consider a square wave kicker, with a 200ns flat top, operating at the bunch frequency.

- **Kicker:** This is conceptually the simplest design, consisting of a magnet which produces a square wave with a 200 ns flat top at the 600 kHz bunch frequency. The field is the minimum required for full extinction as calculated in equation 2 - in this case 14.3 G for a 6 meter magnet. Such a magnet may or may not be practical to build or power.

For this option, the transmission efficiency will just be the fraction of the beam within the 200 ns window, so it is independent of beam emittance and greater than 99% for  $\sigma_t < 35$ ns.

### 3 Magnet Specifications

Table 1 shows the harmonic content for the schemes described above. The magnitudes in the table are the peak integrated bend strengths ( $Bl$ ) relative to the total bend strength required for extinction, as specified in Eq. 2 for  $\beta_x = 250$  m. By definition, the magnitude of the “kicker” option is 1.0.

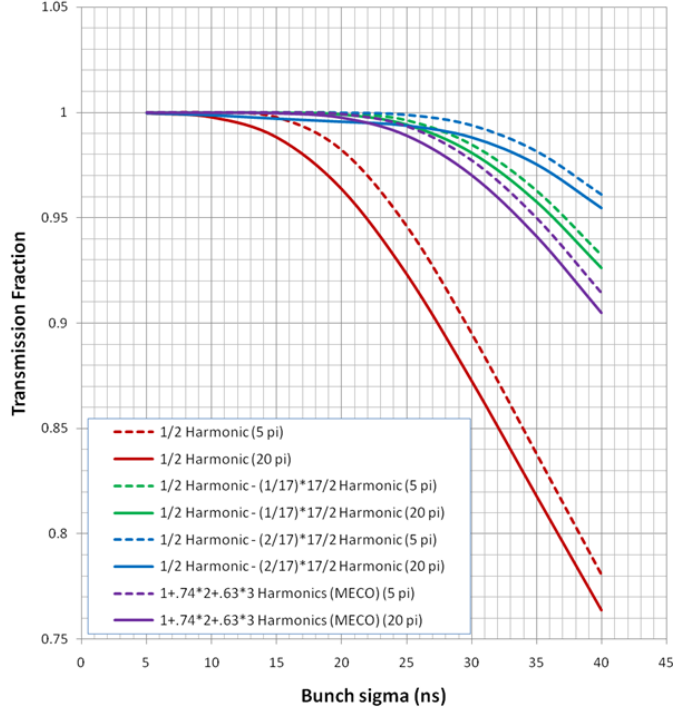


Figure 4: Beam transmission as a function of  $\sigma_t$  for the four AC Dipole configuration. Dashed and solid lines show the results for a 95% normalized emittance ( $\epsilon_{95}$ ) of 5 and 20  $\pi$ -mm-mr, respectively. Gaussian distributions have been assumed in both the longitudinal and transverse planes. In the case of an ideal square wave kicker, the efficiency will be the total fraction of the beam within the 200 ns transmission window.

Configuration	Harmonic Components					
	kHz	Max.	kHz	Max.	kHz	Max.
Sine Wave	300	5.44				
Mod. Sine Wave A	300	5.44	5100	.32		
Mod. Sine Wave B	300	5.44	5100	.64		
MECO Design	600	1.64	1200	1.22	1800	1.03
Kicker	600	1.00				

Table 1: Magnitude the individual frequency components in the configurations considered. Scale is relative to the total required bend field, as specified in Figure 2

. The "kicker" option would be a magnet pulsed with a 200 ns long square wave at the bunch frequency.

The aperture in the bend plane is that required to accommodate the 50  $\pi$ -mm-mr admittance plus the total lateral deflection. In the case of multiple harmonics, we have simply divided the total 6 m length equally amongst the components, although magnet design considerations may lead to a somewhat different allocation of space. This is under the assumption that no beam, even out of time, should hit the magnet. In the case of multiple harmonics, it's assumed that the highest harmonic comes first, to give it the smallest aperture, although this is a minor effect. The aperture in the non-bend plane (magnetic gap) should be as small as possible. As was discussed in [1], if we assume suppression of chromatic effects, the smallest aperture through which a beam may be transmitted is

$$g = 2\sqrt{\frac{AL}{\beta\gamma}} \quad (5)$$

In this case, this would give 11.2 mm. We will assume a full gap width of 1.2 cm for all cases.

For the calculation of the approximate electrical properties, we will assume ideal, single turn, ferrite loaded magnets, such that the magnetic field will be given by:

$$B \approx \mu_0 \frac{I}{g} \quad (6)$$

where  $g$  is the gap width. The inductance will then be

$$L \approx \mu_0 \frac{lw}{g} \quad (7)$$

where  $l$  and  $w$  are the length and width, respectively. For each sinusoidal component, the peak voltage will be given by

$$V_p = 2\pi f L I_p = 2\pi f \left( \mu_0 \frac{wl}{g} \right) \left( \frac{g}{\mu_0} B_p \right) = 2\pi f w l B_p \quad (8)$$

We again assume a gap width 1.2 cm for all the magnets. This leads to the useful approximate relationships shown in Table 2, where we have assumed an aperture in the bend plane of 8 cm when calculating the inductance and peak voltage.

Description	formula
Magnetic Field	$B[\text{G}] = 1.05 \times I[\text{A}]$
Inductance	$L [\mu\text{H}] = 8.4 \times L[\text{m}]$
Peak Voltage	$50.3 \times f[\text{MHz}] \times l[\text{m}] \times B_p[\text{G}]$

Table 2: Useful relationships for the magnets being considered for Mu2e. These calculations assume ideal, single turn, ferrite loaded magnets with an aperture in the bend plane of 8 cm and a gap in the non bend plane of 1.2 cm.

Table 3 summarizes the detailed parameters of the magnets for each of the schemes. In all cases, ideal magnets have been assumed when calculating the inductance and required current. For the case of the kicker, a rise time of 10 ns has been assumed to calculate the peak voltage.

The final column shows the peak stored energy times the frequency. In the case of resonant magnets, one would multiply this by  $2\pi/Q$  to get the total power. This represents the total filling power for the kicker magnet, and might be reduced if some sort of energy recovery is implemented.

In the case of the kicker option, the voltage will be inversely proportional to the rise time. Assuming a single magnet, this a peak voltage of 63.7 kV for the

Config.	Freq. (kHz)	Len. (cm)	Peak Field (G)	Aperture (cm)	Ind. ( $\mu$ H)	Peak Current (A)	Peak Voltage (kV)	$E \times f$ (kW)
<b>Sine Wave</b>	300	600	77.9	8.2	51.5	74.2	7.2	42.6
<b>Mod. Sine A</b>	300	300	155.9	7.8	24.5	148.5	6.9	81.1
	5100	300	9.2	7.3	22.9	8.7	6.4	4.4
<b>Mod. Sine B</b>	300	300	155.9	7.9	24.8	148.5	6.9	82.0
	5100	300	18.4	7.3	23.0	17.5	12.9	17.9
<b>MECO</b>	600	200	70.5	7.7	16.2	67.1	4.1	21.9
	1200	200	52.4	7.5	15.7	49.9	5.9	23.5
	1800	200	44.2	7.3	15.3	42.1	7.3	24.5
<b>Kicker</b>	600	600	14.3	7.4	46.7	13.6	63.7*	2.6

Table 3: Key parameters for all configurations being considered, assuming ideal magnets. The aperture column is the full aperture in the bend plane which is required to preserve the full admittance at peak field. In all cases, the gap in the non-bend plane is 1.2 cm, as determined by the minimum achievable waist. The final column is the peak stored energy times the frequency. For the AC dipoles, this would be multiplied by  $2\pi/Q$  to get the actual power. In the case of the kicker, it would be multiplied by the fractional energy loss per cycle. (\*assuming a rise time of 10 ns.)

## 4 Discussion

The simple, single harmonic in the Mu2e proposal will probably have unacceptable losses for realistic bunch lengths. In addition, the high slew rate the transmission time will probably require a compensating dipole. In the case of the other three schemes, it is likely that the compensating magnet will not be required.

Given the difficulty of designing high frequency magnets, the minor transmission improvement of “Modified Sine Wave B” almost certainly doesn’t warrant the doubling of the required field for the 5.1 MHz component.

Therefore, the most promising candidates at the moment appear to be the “Modified Sine Wave A” scheme and the ”MECO” scheme, and should be made based on the difficulty of designing the highest frequency magnet. Specifically,

- **Modified Sine Wave A:**

Frequency: 5.1 MHz

Peak Field: 9.2 Gauss

Peak Current: 8.7 A

Length: 3 m

Inductance: 22.9  $\mu$ H

Peak Voltage: 6.4 kV

- **MECO:**

Frequency: 1.8 MHz

Peak Field: 44.2 Gauss

Peak Current: 42.1 A

Length: 2 m

Inductance: 15.3  $\mu$ H

Peak Voltage: 7.3 kV

If these turn out to be a similar challenge, then the first scheme has the advantage of requiring only two types of magnet rather than three.

It has been assumed throughout that the simple kicker option would be impractical, requiring resonant (AC dipole schemes). However recent advances in high power broad band technology may change this and make the option at least worth discussing. The parameters for such a magnet are approximately as follows:

- **Kicker**

Repetition rate: 600 kHz

Flat Top: 200 ns

Rise Time: order of 10 ns

Peak Field: 14.3 Gauss

Peak Current: 13.6 A

Length: 6 m

Inductance: 46.7  $\mu$ H

Peak Voltage: 63.7 kV (for 10 ns rise time)

The high voltages resulting from a fast rise time may prove very challenging, but as with all magnets, the option exists to break the kicker into shorter segments.

## References

- [1] E. Prebys, “Optimizing of AC Dipole Parameters for Beam Extinction”, <http://mu2e-docdb.fnal.gov>, Mu2-doc-534-v1
- [2] *Carol Johnstone, private communication.*
- [3] E. Prebys, “Parametric Analysis of Beam Transmission in the Mu2e Extinction Channel”, <http://mu2e-docdb.fnal.gov>, Mu2-doc-550-v3
- [4] Mu2e Collaboration, “Proposal to Search for  $\mu N \rightarrow e N$  with a Single Event Sensitivity Below  $10^{-16}$ ”, FNAL Proposal E-973, <http://mu2e-docdb.fnal.gov>, Mu2e-doc-388-v1. Sec. 6.5.
- [5] W. Molzon, “Proton Beam Extinction”, MECO-EXT-05-002 (2005), [http://meco.ps.uci.edu/old/ref\\_design/MECO-EXT-05-001V1.02.pdf](http://meco.ps.uci.edu/old/ref_design/MECO-EXT-05-001V1.02.pdf)